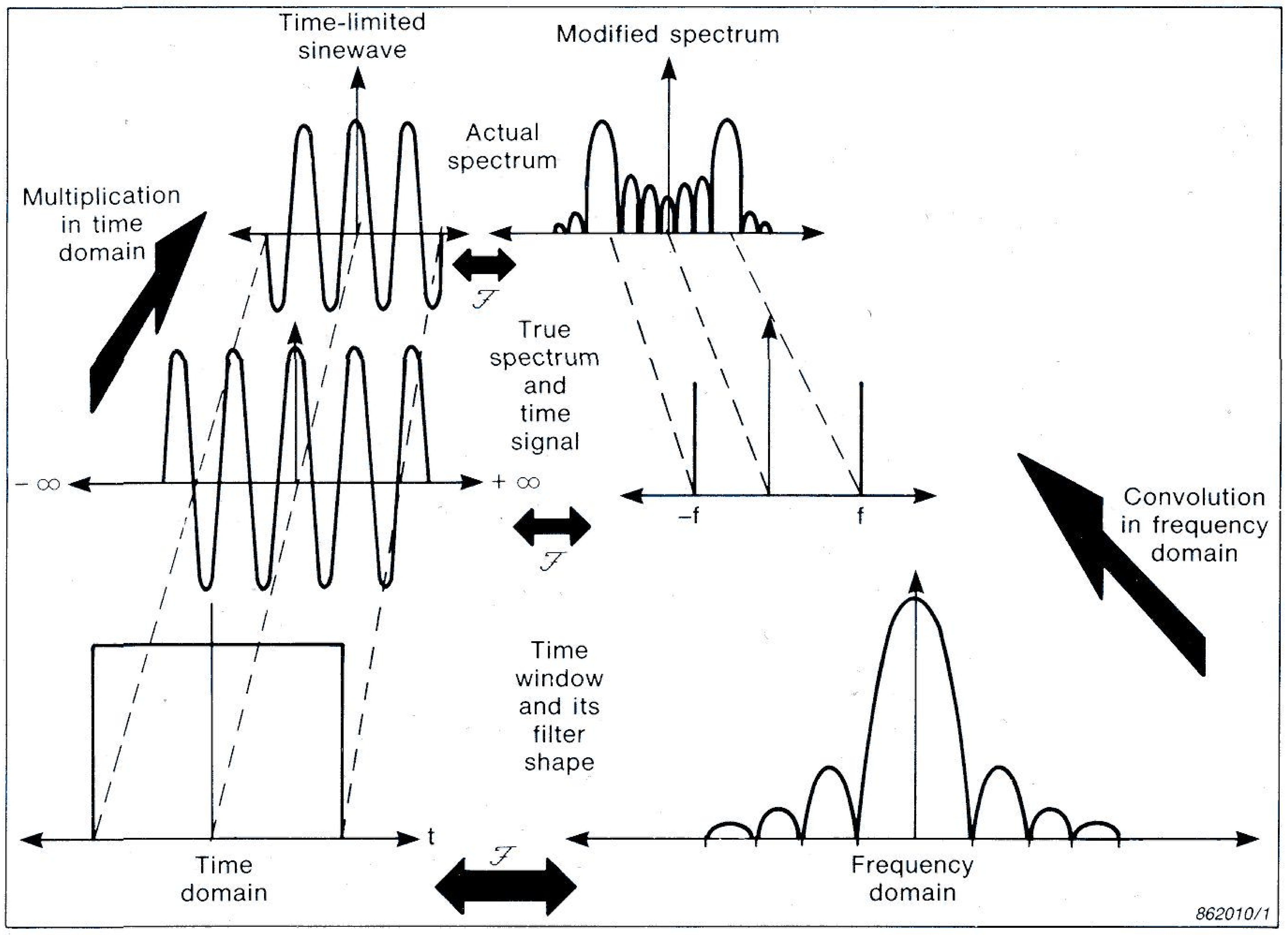
Brüel & Kjær application notes

Time Windows

FFT analyzers such as the Brüel & Kjær Dual Channel Analyzers Types 2032 and 2034 process finite blocks of digital information. The analyzer extracts, or "cuts out" these blocks from the complete time record using time weighting functions, or "time windows". Unless we can understand the basic concepts of time-limited data, we can't be absolutely sure that our interpretation of FFT analysis is correct. Neither can we be sure of the accuracy of our results.



FFT analyzers view time signals through windows and so change the frequency spectrum

What do time weighting functions do?

Fourier analysis tells us that time and frequency are simply two alternative ways of observing a signal. By changing the nature of a signal in the time domain, we implicitly change the nature of the spectrum in the frequency domain. This is exactly what we do when we apply a weighting function, or "time window", to a signal. In the case of continuous signals, time windows "slice" the data into sections which the analyzer then analyzes. In the case of transient data, time windows edit the time record so the analyzer only works on the transient, and not the portions of data containing only noise, before and after the transient. However, by taking such slices of the original time domain data we have changed, or filtered, the corresponding spectrum of our data in the frequency domain.

The type of weighting, or filtering, in the frequency domain is determined by the shape of the window through which the analyzer "sees" the data in the time domain. There are many types of window. The choice of window depends on the type of signal being analyzed and, consequently, the application.

Implementing windows

The analyzer simply multiplies the window and the signal together in the time domain. Alternatively, the analyzer could use a convolution of the spectra in the frequency domain.

How can we quantify their effects?

We quantify the effect in the frequency domain. The simplest way is to consider weighting functions as filters and use traditional filter descriptors.

Effective Noise Bandwidth is the width of an ideal filter with the same transmission level, and which transmits the same power from a white noise source.

3dB Bandwidth is the distance in Hertz or Radians per second between the half-power (–3dB) points on the amplitude axis.

The bandwidth of a filter tells us how well a filter can separate frequency components of similar level.

Selectivity tells us how well a filter separates components of very different levels. We call the ratio of a filter's –3dB Bandwidth to its –60dB the Shape Factor and this is the most basic measure of selectivity.

"Ripple" appears in the pass band of a filter. We measure, in dBs, the height of the ripple within ± 1/2 line spacing around the centre frequency.

Noise Bandwidth and Selectivity tell us how well a filter determines the

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frequency content of a signal, whereas the ripple determines the accuracy of the amplitude of a filtered signal.

A choice of weighting functions

The 2032 and 2034 have 7 weighting functions. Four of these windows, i.e. Hanning, Rectangular, Kaiser-Bessel and Flat-top, have fixed characteristics. The other three, i.e. Transient, Exponential and User-defined, can have their properties controlled by the user.

The type of measurement we make determines the window we use.

Fixed Windows

Rectangular Window

This window, otherwise known as the "Flat Window" or "Box-Car Window", is not really a weighting at all. It simply has a unity value within the record length, T, of the analyzer and zero value outside.

$w(t) = 1$ for $0 \leqq t < T$

$w(t) = 0$ elsewhere

This window simply cuts the data when the record length of the analyzer is reached. In the case of, for example, a sine-wave, discontinuities at the start and end of the time-limited data cause "leakage" of energy from the main frequency of the sine-wave into adjacent frequencies.

The Fourier Transform of the window gives the filter shape shown. Because all filters in a Discrete Fourier Transform (DFT) Analyzer show symmetry around the centre frequency, only the right hand side needs to be shown.

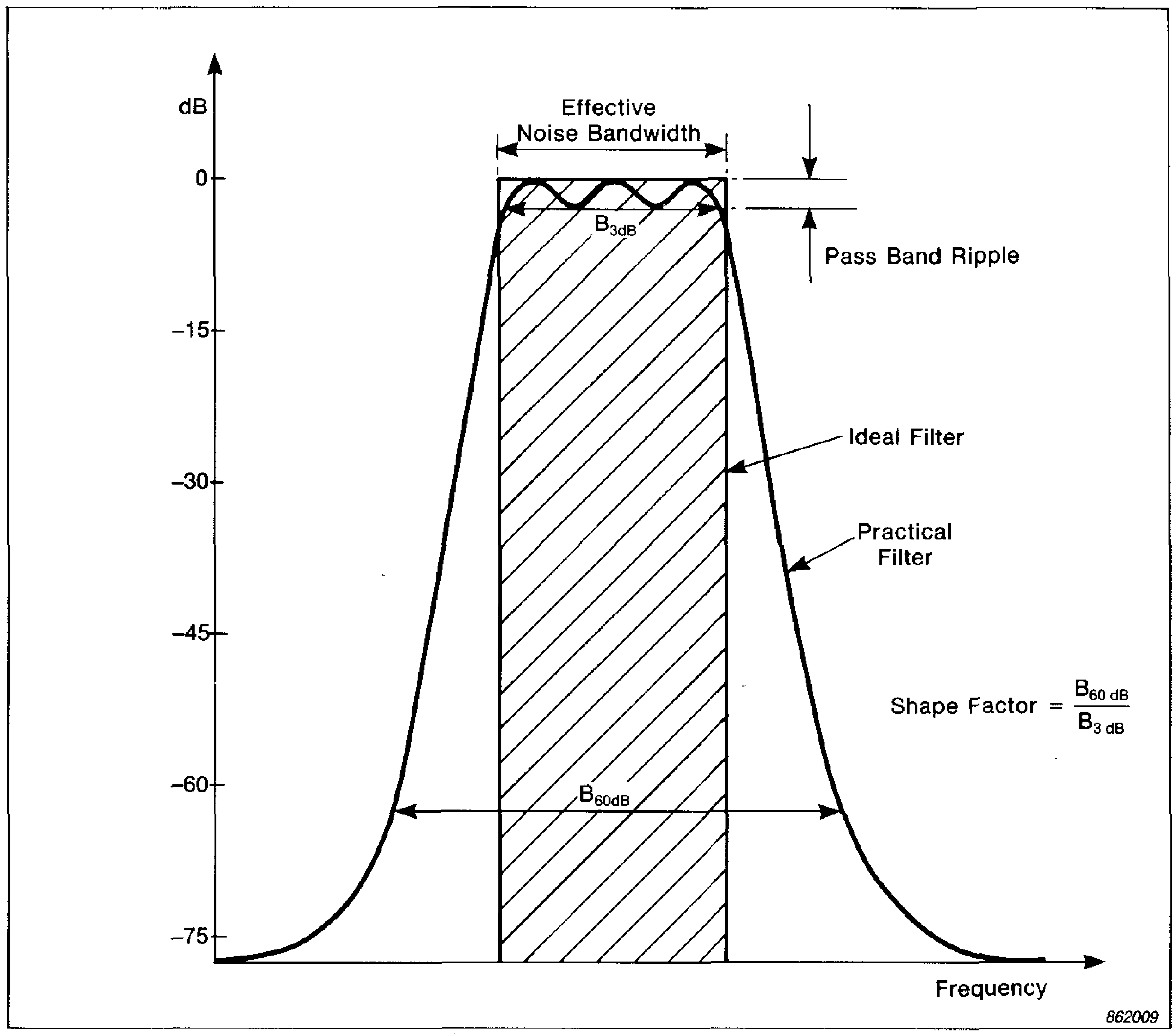

Fig. 1. The definitions of filter parameters

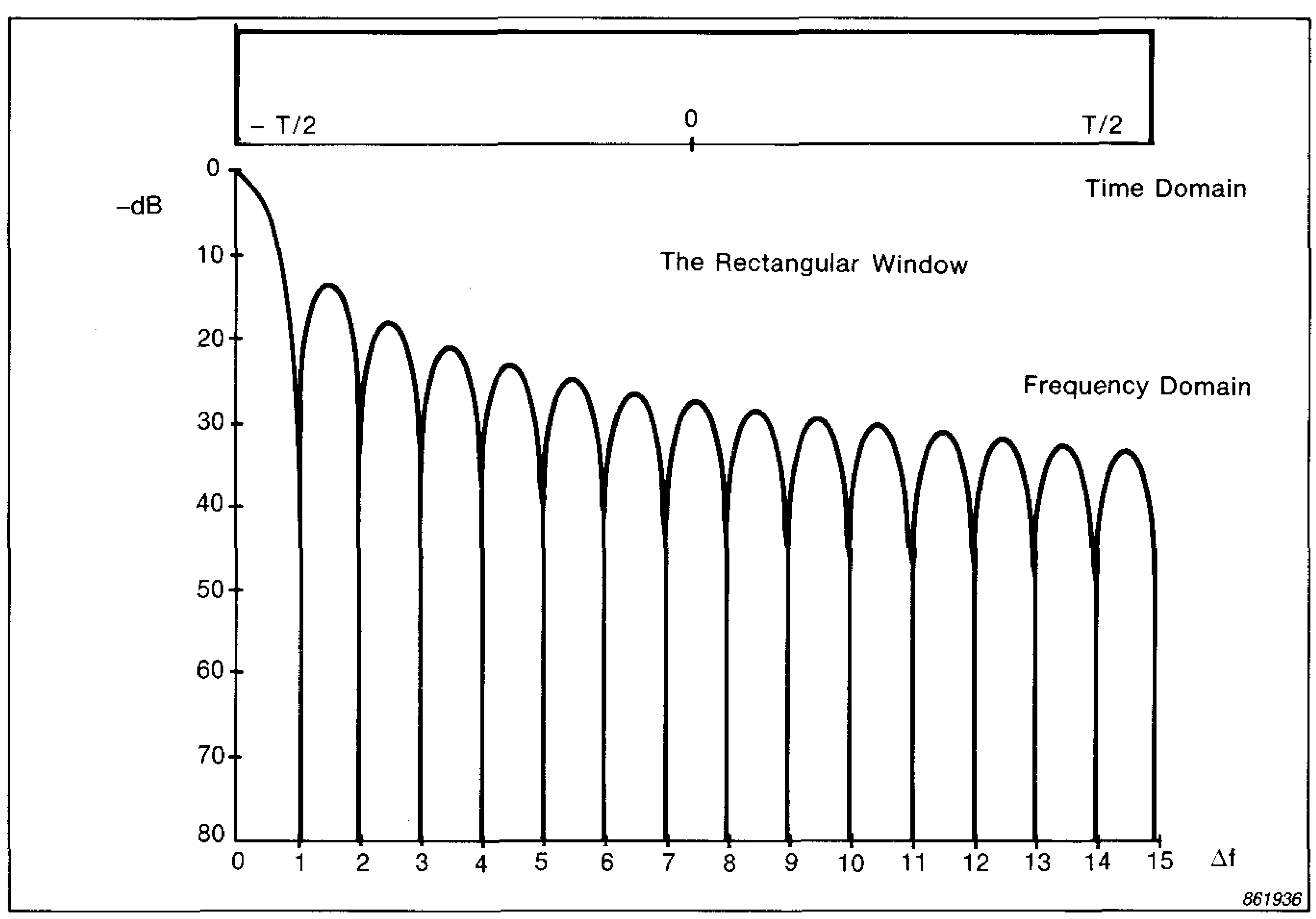

Fig. 2. Frequency and time characteristics of the rectangular window

The filter exhibits a main lobe followed by a series of smaller lobes. The main lobe has a width equal to twice the line-spacing Δf. The first lobe is attenuated 13 dB relative to the main lobe and the side lobe fall-off rate is 20 dB per decade. Consequently, the selectivity of the weighting is very poor and there is a large amount of ripple in the pass-band.

Let us apply this window to a sinusoidal signal. Take the case where the frequency of the sine wave lies exactly between two lines on the frequency axis. The loss in power from the main line to the rest of the 800 lines causes an amplitude under-estimation of 3.9 dB. The spectrum of the truncated sine wave has a similar shape to that of the filter. The spectrum is sampled twice in the main lobe and at every peak in the smaller lobes.

When the frequency of the sinewave coincides exactly with an analysis line, the spectrum is sampled in the centre of the main lobe and only at zero crossings of the side-lobes. The amplitude errors are zero. This is a "best case" situation. The Effective Noise Bandwidth of the weighting is equal to the line spacing Δf.

This type of window is a poor one to use on continuous signals. However, if the signal is synchronized to the record length we can achieve the "best case" situation above. We do this in order analysis and mobility measure-